Towards a Determination of the Lowest Moments of Nucleon Structure Functions in Full QCD

QCDSF/UKQCD

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Outline

- Review of the problem
- Computational details
- Discussion of the results
- Conclusions and outlook

Moments of Unpolarised Nucleon Structure Functions

$$\int_0^1 dx\, x^{n-2}\, F^{\rm NS}(x,Q^2) = f\, E^{\mathcal{S}}_{F;v_n}\left(\frac{M^2}{Q^2}, g^{\mathcal{S}}(M)\right) v_n{}^{\mathcal{S}}(g^{\mathcal{S}}(M))$$

- Matrix elements v_n can be measured on the lattice

$$\langle N(\vec{p})| \left[\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} - \mathrm{Tr}\right] |N(\vec{p})\rangle^{\mathcal{S}} := 2v_n^{(q)\mathcal{S}}\ [p^{\mu_1}\cdots p^{\mu_n} - \mathrm{Tr}]$$

- Results have to be renormalised using a scheme (e.g. $\overline{\mathrm{MS}}$) and scale (e.g. 2 GeV).

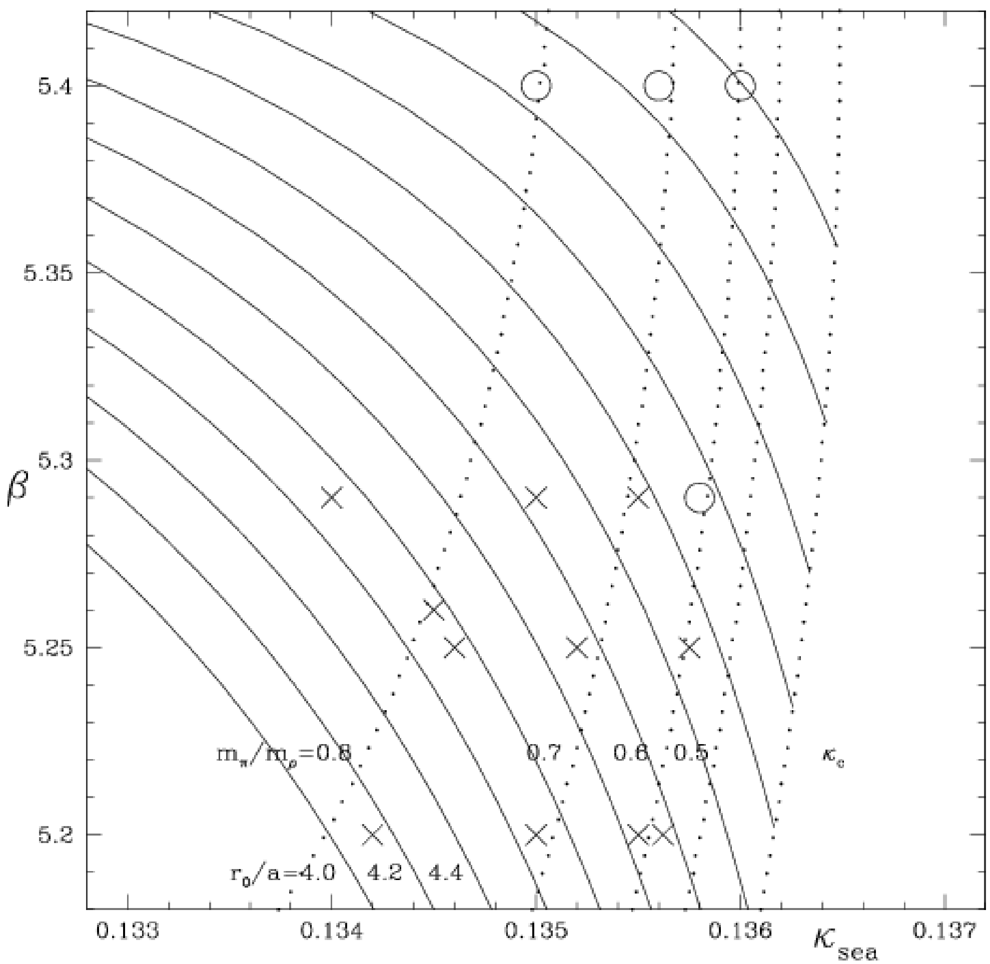
- Lattice data needs to be extrapolated to chiral and continuum limit.

Simulations Parameters (1)

Configurations with $N_\mathrm{f} = 2$ O(a)-improved dynamical quarks generated by UKQCD+QCDSF:

$(\beta, \kappa_\mathrm{sea})$	$m_\mathrm{PS}/m_\mathrm{V}$	a [fm]	L_s [fm]	#trajectories
(5.20,0.13420)	0.789(2)	0.123	2.0	O(5000)
(5.20,0.13500)	0.711(4)	0.105	1.7	O(8000)
(5.20,0.13550)	0.602(6)	0.099	1.6	O(8000)
(5.25,0.13460)	0.784(2)	0.106	1.7	O(5800)
(5.25,0.13520)	0.719(4)	0.097	1.6	O(8000)
(5.25,0.13575)	0.613(5)	0.091	2.2	O(5900)
(5.26,0.13450)	0.789(4)	0.106	1.7	O(4000)
(5.29,0.13400)	0.833(2)	0.104	1.7	O(4000)
(5.29,0.13500)	0.759(2)	0.096	1.5	O(5600)
(5.29,0.13550)	0.702(5)	0.090	2.2	O(2000)
(5.40,0.13500)	0.802(2)	0.082	2.0	O(3700)
(5.40,0.13560)	0.731(3)	0.079	1.9	O(3500)
(5.40,0.13610)	0.629(11)	0.075	1.8	O(2400)

Simulation Parameters (2)



Calculation of Matrix Elements (1)

Using the following operators (for $\vec{p} = (1,0,0)$)

$$
\begin{aligned}
\mathcal{O}_{v_{2a}} &= \mathcal{O}^{\gamma}_{\{14\}} \\
\mathcal{O}_{v_{2b}} &= \mathcal{O}^{\gamma}_{\{44\}} - \frac{1}{3}\left(\mathcal{O}^{\gamma}_{\{11\}} + \mathcal{O}^{\gamma}_{\{22\}} + \mathcal{O}^{\gamma}_{\{33\}}\right) \\
\mathcal{O}_{v_3} &= \mathcal{O}^{\gamma}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\gamma}_{\{224\}} + \mathcal{O}^{\gamma}_{\{334\}}\right) \\
\mathcal{O}_{v_4} &= \mathcal{O}^{\gamma}_{\{1144\}} + \mathcal{O}^{\gamma}_{\{2233\}} - \mathcal{O}^{\gamma}_{\{1133\}} - \mathcal{O}^{\gamma}_{\{2244\}}
\end{aligned}
$$

where

$$
\mathcal{O}_{q;\mu_1\cdots\mu_n} = \bar{q}\,\Gamma_{\mu_1\cdots\mu_i}\,\overleftrightarrow{D}_{\mu_{i+1}}\cdots\overleftrightarrow{D}_{\mu_n}\,q
$$

v_{2a} and v_{2b} → different representations of same continuum operator

Calculation of Matrix Elements (2)

Matrix elements are calculated from

$$\mathcal{R}(t,\tau;\vec{p};\mathcal{O}) \quad = \quad \frac{C_{\frac{1}{2}(1+\gamma_4)}(t,\tau;\vec{p};\mathcal{O})}{C_{\frac{1}{2}(1+\gamma_4)}(t;\vec{p})}$$

using (for $\vec{p}=(1,0,0)$)

$$\begin{aligned}
\mathcal{R}(t,\tau;\vec{p};\mathcal{O}_{v_{2a}}) &= ip_1\, v_{2a} \\
\mathcal{R}(t,\tau;\vec{p};\mathcal{O}_{v_{2b}}) &= -\frac{E_{\vec{p}}^2+\frac{1}{3}\vec{p}^2}{E_{\vec{p}}}\, v_{2b} \\
\mathcal{R}(t,\tau;\vec{p};\mathcal{O}_{v_3}) &= -p_1^2\, v_3 \\
\mathcal{R}(t,\tau;\vec{p};\mathcal{O}_{v_4}) &= E_{\vec{p}}\, p_1^2\, v_4
\end{aligned}$$

Simulation Details

Optimise use of expensive dynamical configurations:

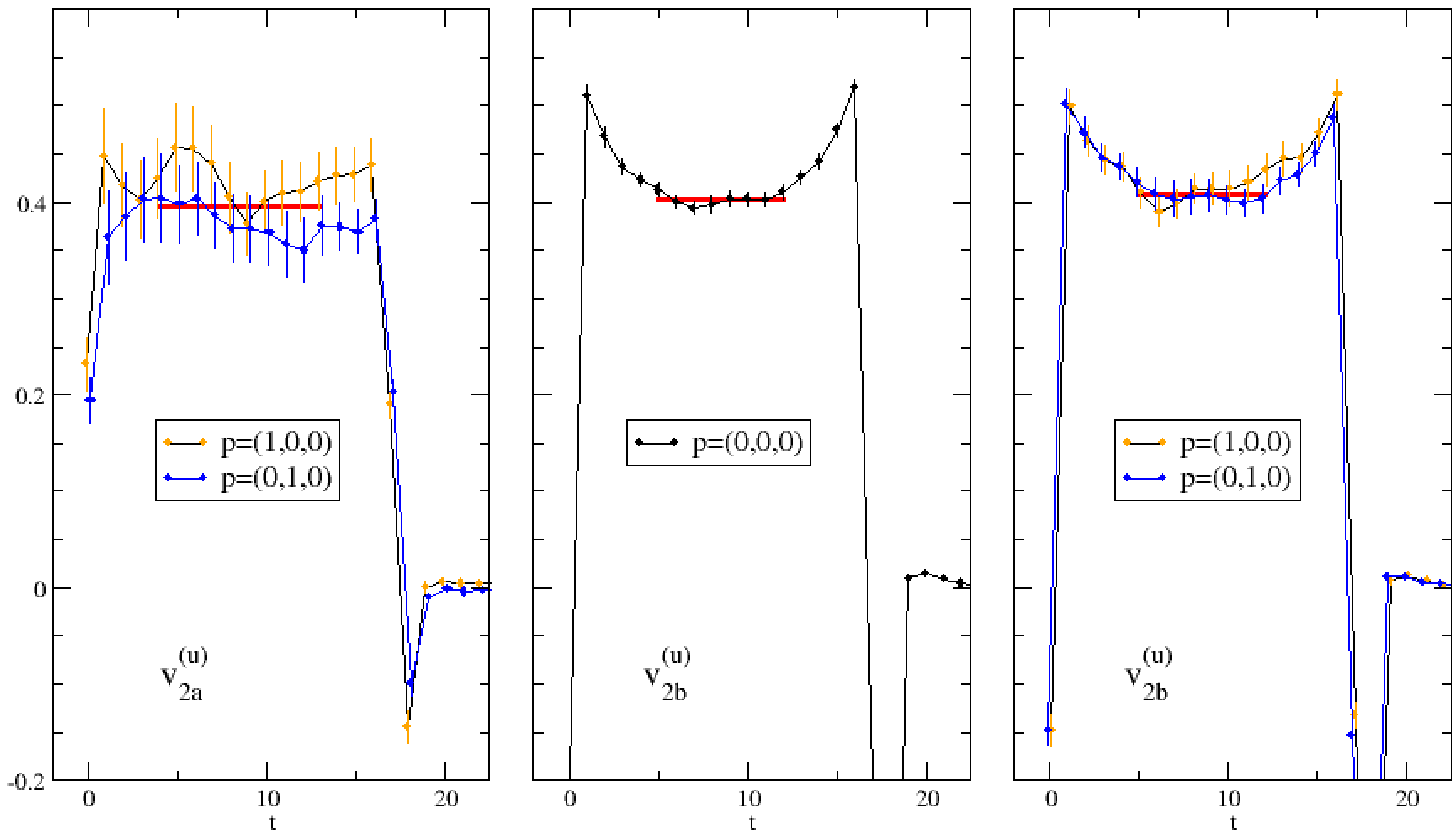
- ❑ Take configurations at distance 5-10 trajectories, but use 8-4 different locations of the source

- ❑ Use two different momenta:

$$\vec{p} = (\quad 1, \quad 0, \quad 0 \quad)$$
$$\vec{p} = (\quad 0, \quad 1, \quad 0 \quad)$$

→ Use binning to eliminate auto-correlations

Example: v_{2a} vs. v_{2b}



$(\beta, \kappa_{\text{sea}}) = (5.4, 0.13560)$, V=$24^3 \times 48$

Operator Improvement and Mixing

- O(a)-Improvement requires operators to be improved, e.g. $v_{2\mathrm{a}}$ or $v_{2\mathrm{b}}$:

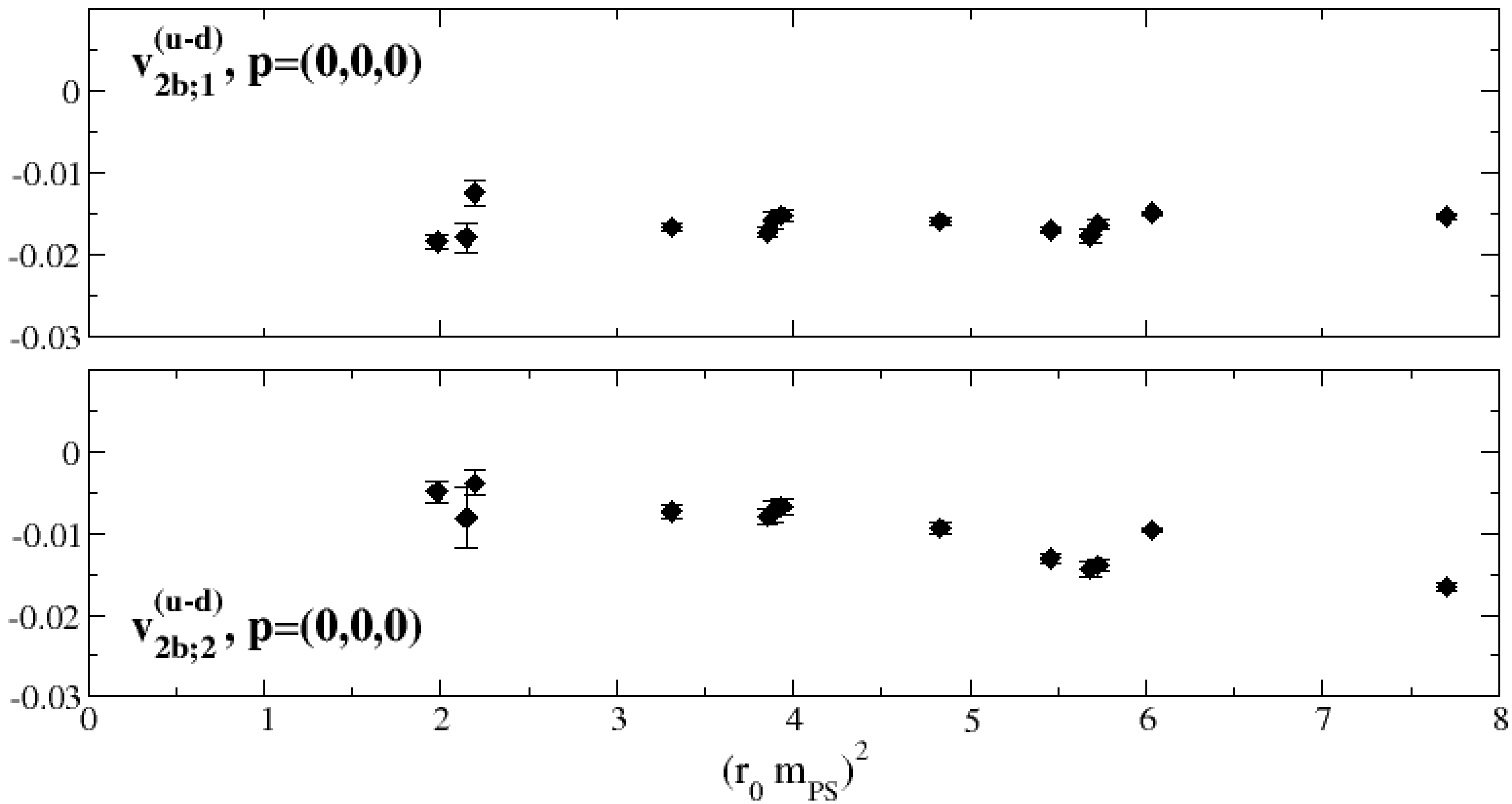
$$\mathcal{O}^{\gamma}_{\mu\nu} \to (1 + a m_{\mathrm{q}} c_0)\, \mathcal{O}^{\gamma}_{\mu\nu} + \sum_{i=1}^{2} a\, c_i\, \mathcal{O}^{(i)}_{\mu\nu}$$

Relations between some of the improvement coefficients are known perturbatively.

- Operators $\mathcal{O}_{v_3}$ and $\mathcal{O}_{v_4}$ can mix with relevant operators

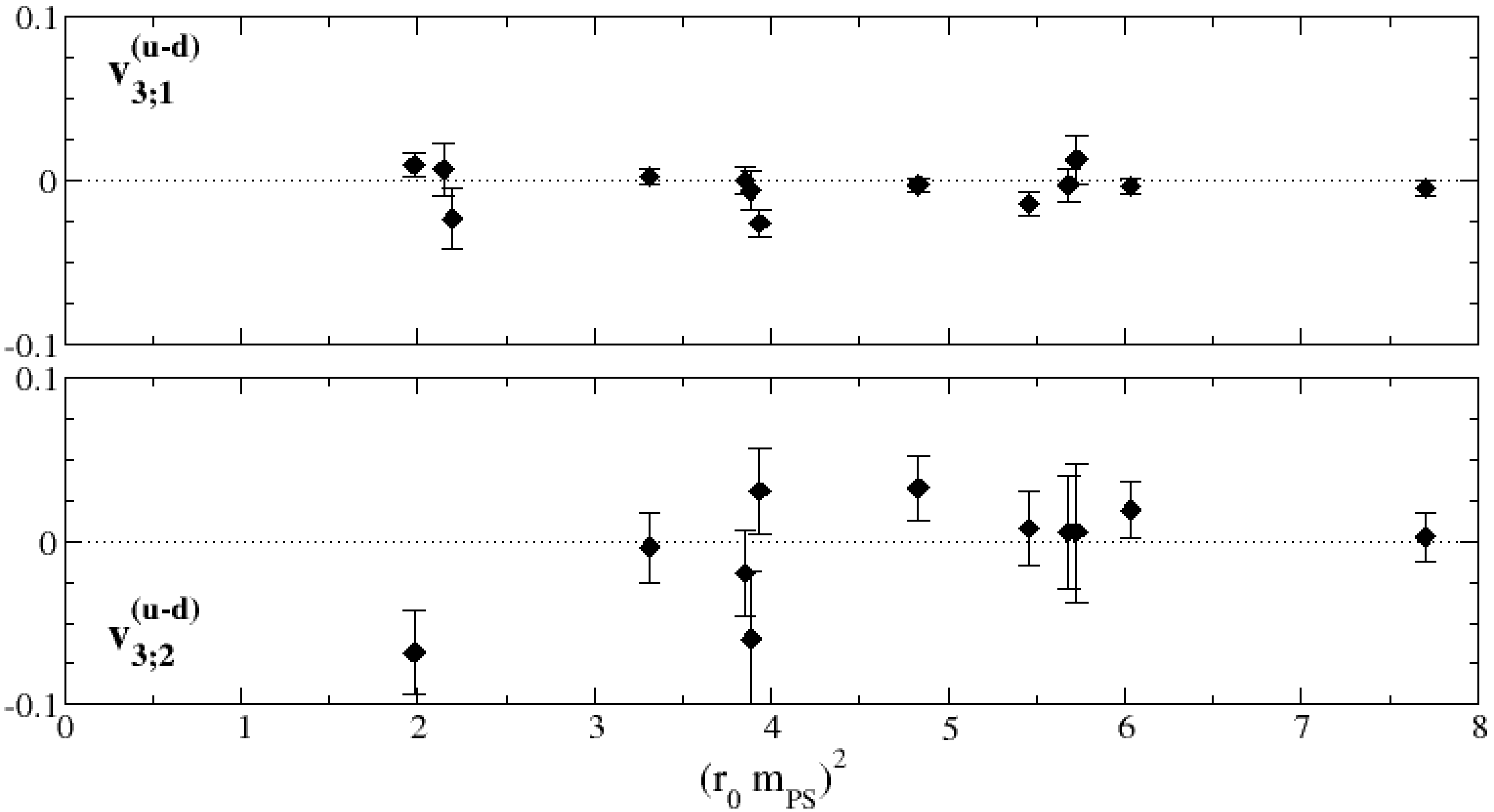
Improvement Terms

Improvement coefficients c_1 and c_2 expected to be small.



$\rightarrow$ Use $c_1 = c_2 = 0$ and perturbative results for c_0.

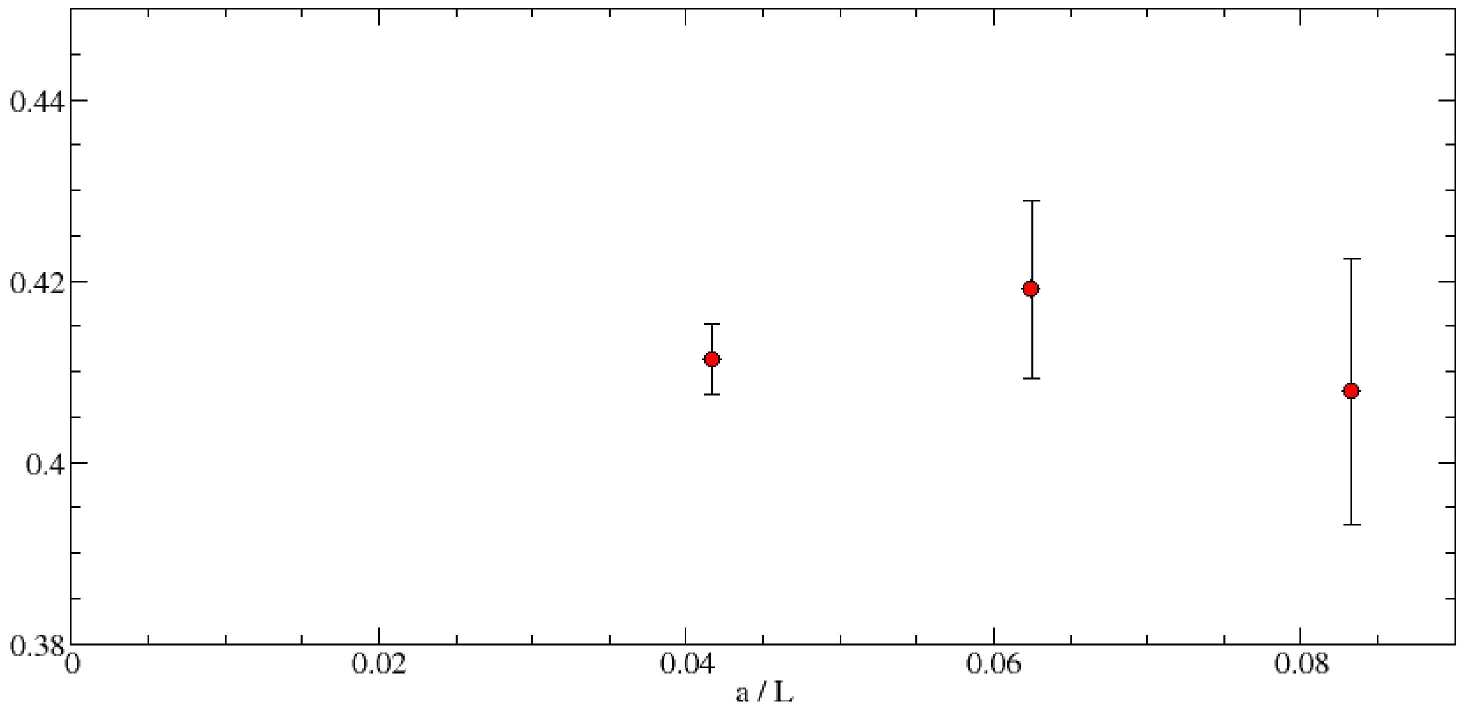
Mixing Terms



→ Mixing negligible (similar result for v_4)

Finite Size Effects?

E.g., v_{2b} $(\vec{p} = (0,0,0))$ at $(\beta, \kappa_{\rm sea}) = (5.29, 0.13550)$ on
$V = 24^3 \times 48$, $16^3 \times 32$, $12^3 \times 32$:



$\longrightarrow$ Finite size effects small

Renormalisation Strategy

(lattice, a)

Translate lattice results to RGI $\Downarrow$ $\Delta Z_{\mathcal{O}}^{\mathrm{lat}}(a)$

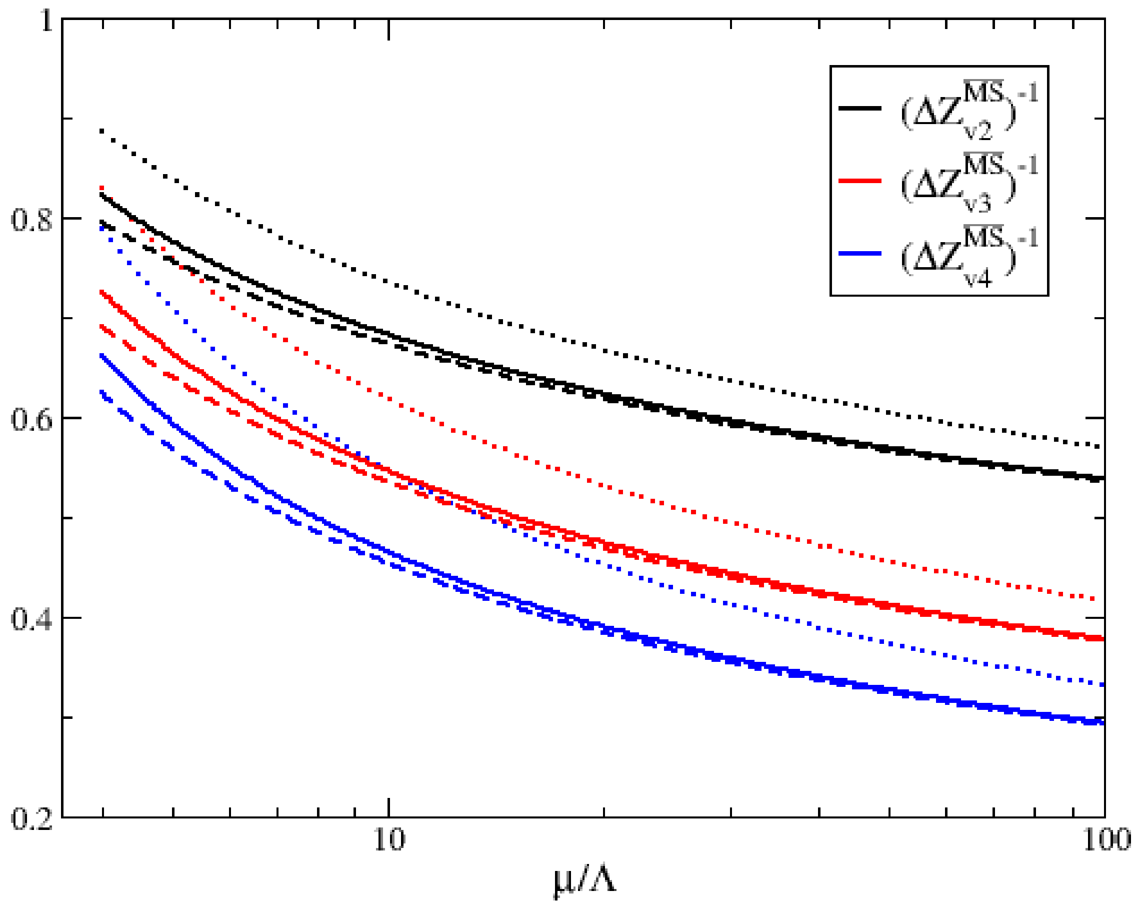
RGI

Translate RGI into $\overline{\mathrm{MS}}$ $\Downarrow$ $\left(\Delta Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}(\mu = 2\mathrm{GeV})\right)^{-1}$

($\overline{\mathrm{MS}}$, 2GeV)

Renormalisation: $\overline{\mathrm{MS}}$

β-function and γ-function for $\mathcal{O}_{v_\mathrm{i}}$ (i=1,2,3) known to ≥ 3 loops:



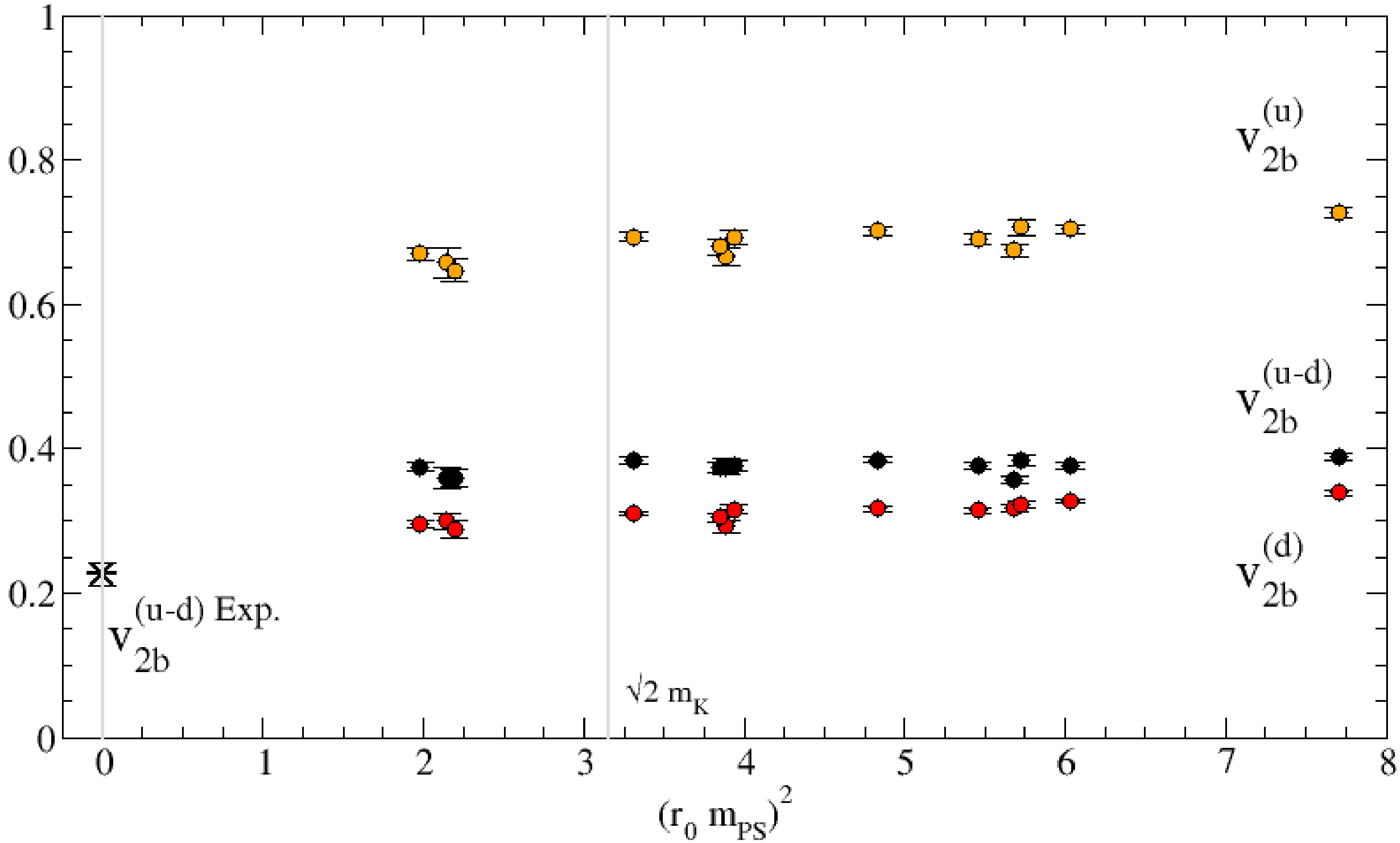
Renormalisation: Lattice

Lattice renormalisation constant known upto 1-loop.

Use tadpole-improved, renormalisation-group-improved, boosted perturbation theory:

$$\Delta Z_{\mathcal{O}}^{\mathrm{lat}}(a) = u_0^{1-n_D}\left[2b_0 g_\square^2\right]^{\frac{d_{\mathcal{O};0}}{2b_0}}\left[1+\frac{b_1}{b_0}g_\square^2\right]^{\frac{b_0 d_{\mathcal{O};1}^{\mathrm{lat}}-b_1 d_{\mathcal{O};0}}{2b_0 b_1}+\frac{p_1}{4}\frac{b_0}{b_1}(1-n_D)}$$

Results for v_{2b}^{RGI}



Chiral and Continuum Extrapolation

Noisy data spoils attempts to separate extrapolations
➞ attempt simultaneous chiral and continuum extrapolation

Ansatz:

$$v_n^{\rm RGI}(r_0, m_{\rm PS}) = \underbrace{F^{v_n}(r_0 m_{\rm PS})}_{\text{chiral extr.}} + \underbrace{c_n \left(\frac{a}{r_0}\right)^2}_{\text{cont. extr.}} + \underbrace{d_n\, a r_0\, m_{\rm PS}^2}_{\propto\, a\, m_{\rm q}}$$

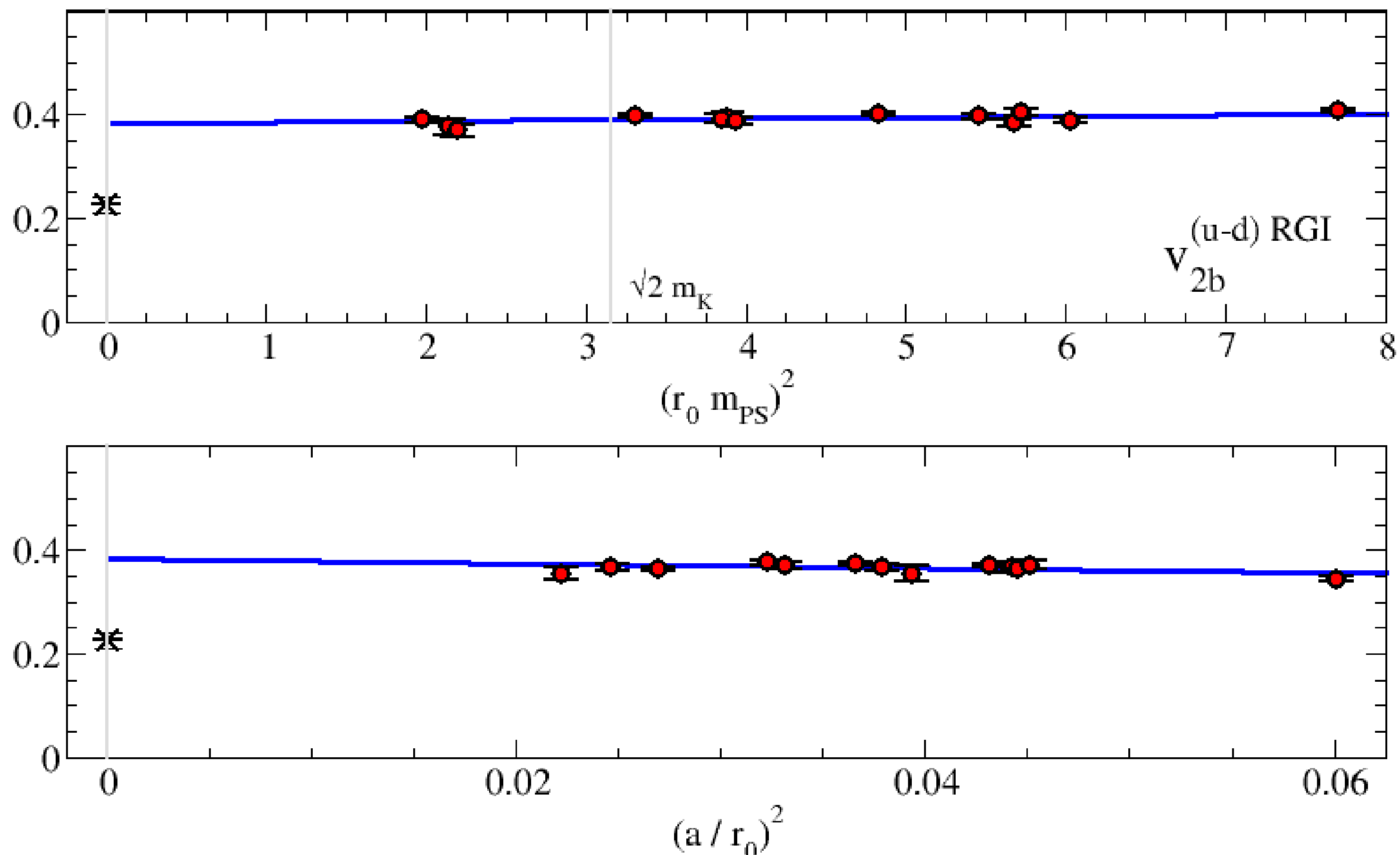
where [Thomas et al., Detmold et al.]

$$F^{v_n}(x) = v_n^{\rm RGI}\left(1 - C x^2 \ln \frac{x^2}{x^2 + (r_0 \Lambda_\chi)^2}\right) + a_n x^2$$

$C \approx 0.663$, for $\Lambda_\chi = 0$ ➞ extrapolation linear in quark mass
We use: $d_n = 0$

Linear Extrapolation

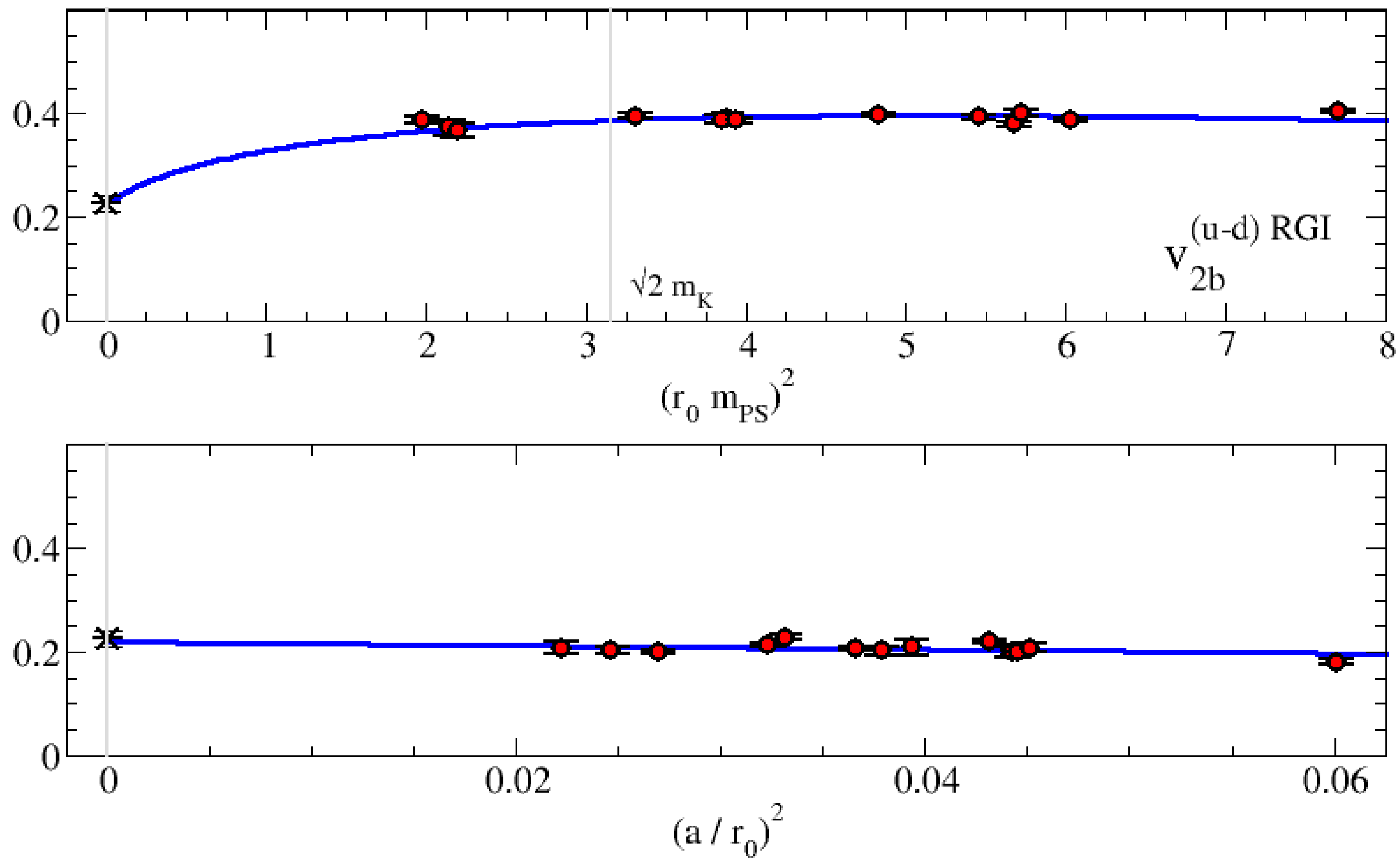
$v_{2b}^{\mathrm{RGI}}(r_0, m_{\mathrm{PS}}) - c_{2b}(a/r_0)^2$



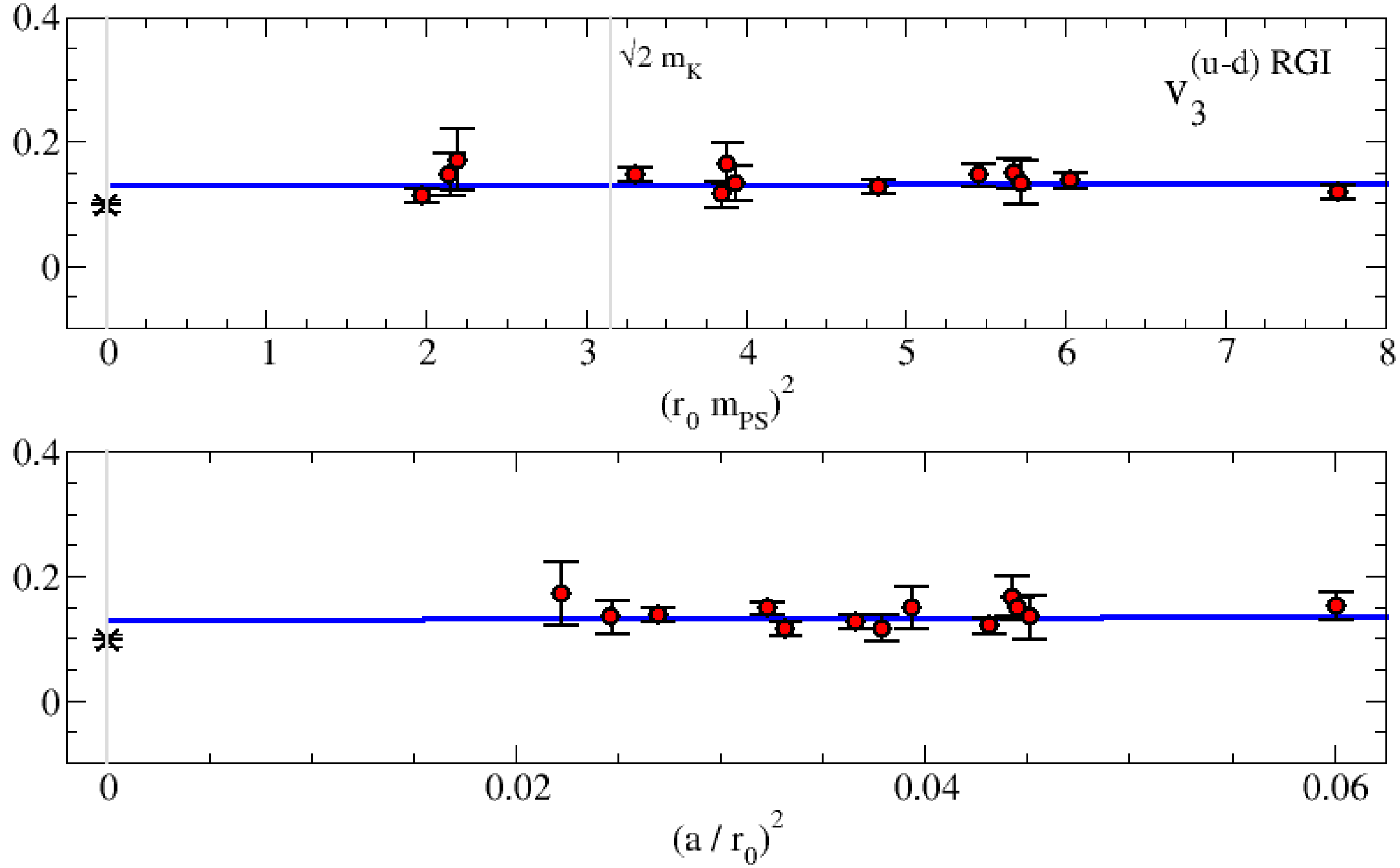
$v_{2b}^{\mathrm{RGI}}(r_0, m_{\mathrm{PS}}) - a_{2b}(r_0 m_{\mathrm{PS}})^2$

Does Lattice Data Disagree with Experiment?

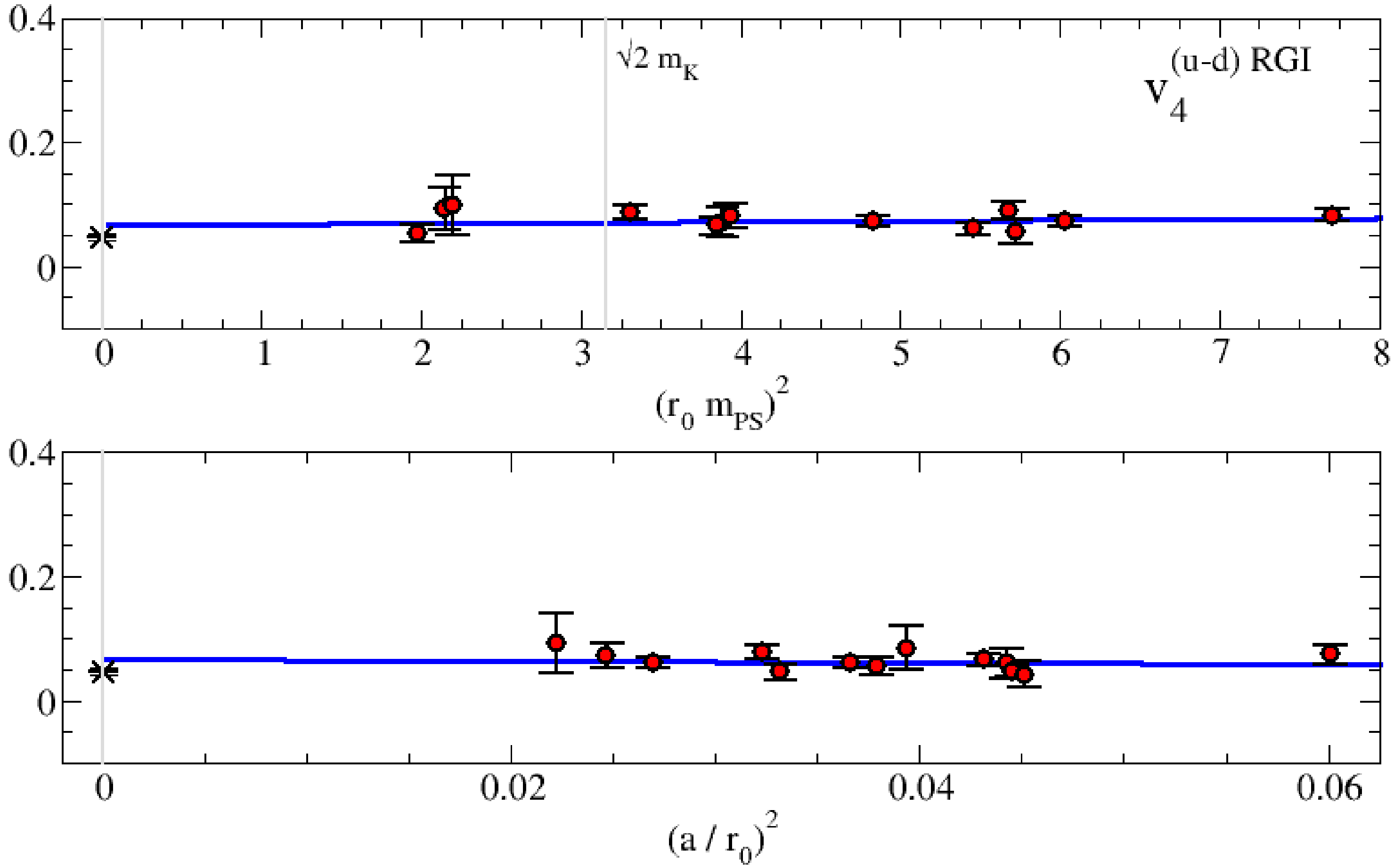
Fit with $\Lambda_\chi = 1\mathrm{GeV}$ and v_2 fixed to experimental value



Results for $v_3^{(\mathrm{u-d})\mathrm{RGI}}$



Results for $v_4^{(u-d)\mathrm{RGI}}$



Conclusions and Outlook

- Analysis of current data for $N_\mathrm{f} = 2$ O(a)-improved Wilson-fermions render very similar results as for $N_\mathrm{f} = 0$.

- While data at lighter sea quark masses closer to continuum became available, extrapolation of the lattice results remains a problem.

- Data in region where chiral perturbation theory might apply have become available, but higher statistical accuracy needed.

- NP-renormalisation is work-in-progress.